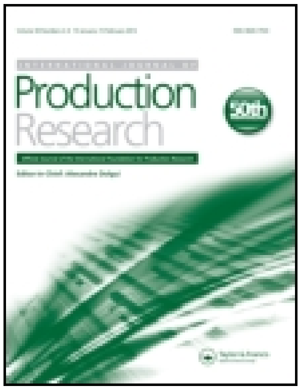

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On the integer programming formulation of production scheduling optimisation algorithm for the hot rolling processes

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This note is concerned with the formulation of scheduling of the hot rolling process (SHRP). Based on the capacitated vehicle routing problem (CVRP), Chen *et al*. (Chen, A.L., Yang, G.K., and Wu, Z.M., 2008. Production scheduling optimization algorithm for the hot rolling processes. *International Journal of Production Research*, 46 (7), 1955–1973) proposed a nonlinear integer programming formulation of SHRP. Due to some deficiencies in the formulation, Kim (Kim, B.-I., 2010. Some comments on Chen *et al*. 'Production scheduling optimization algorithm for the hot rolling processes'. *International Journal of Production Research*, 48 (7), 2165–2167) very recently gave some correction to the model. However, even with the correction the model has flaws. The purpose of this note is to give a complete, also based on CVRP, corrected formulation with substantial number of variables reduced.

Keywords: hot rolling production scheduling; vehicle routing problem

Chen *et al*. (2008) recently proposed a nonlinear programming model for production scheduling of the hot rolling processes (SHRP). They formulated SHRP as the capacitated vehicle routing problem (CVRP). In a very recent paper, Kim (2010) showed some major flaws in the model of Chen *et al*. (2008) and gave corrections. Here we show that although the correction by Kim (2010) is an improvement over the original formulation, it is still not sufficient to fix the problem. The purpose of this note is to present a correction to the CVRP model of Chen *et al*. Our formulation is also the first publication to include a lower and upper bound to CVRP in the general case, although Kara and Bektas (2006) have presented a special case. The formulation by Chen *et al*. includes $K(2N^2+3N+1)$ binary variables, and the formulation by Kim includes $K(N^2+2N+1)$ binary variables. Our CVRP formulation reduces the number of variables to N^2+N binary, and $2N$ continuous variables for subtour elimination. The formulations by Chen *et al*. (2008) and Kim (2010) require subtour elimination that is not present in their models. With added subtour elimination to their models, the number of variables in each case increases additionally by $2N$ continuous variables. We refer to Chen *et al*. (2008) for a good explanation of SHRP.

We are given a set of N orders (slabs) to be processed by a set of K turns (machines). The rolling length of the order i is equal to q_i. Several conditions should be satisfied:

(1) The orders should be rolled from wide to narrow.
(2) The order width jump should be small, and order thickness and hardness should not be allowed to jump repeatedly and should change smoothly. The order temperature should also change smoothly.
(3) The width, thickness, and hardness should not be allowed to jump simultaneously. When the three items compete against each other, the order of priority is: hardness, thickness, and width.
(4) The total length of the come-down body is limited to a given value.
(5) The total length of the continuously rolled orders with the same width is limited to a given value.

Let Q be the length limit of continuously rolled slabs with the same width; $L(U)$ the lower (upper) limit of the rolling total length; and C_{ij} a given penalty calculated based on value of width, thickness, hardness, and temperature jumps. The objective of SHRP is to minimise the total penalty caused by width, thickness, hardness and temperature jumps between adjacent orders. Chen *et al*. (2008) showed that SHRP can be solved using a CVRP. They gave

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a CVRP formulation of the SHRP model as follows:

$$\text{Minimise} \quad Z1 = \sum_{k=1}^{K}\sum_{i=0}^{N}\sum_{j=0}^{N} C_{ij}x_{ij}^{k}$$

$$\text{s.t.} \quad \sum_{j=0}^{N} x_{ij}^{k} = y_{i}^{k}, \quad i = 0, 1, \ldots, N,\ k = 1, \ldots, K, \tag{1}$$

$$\sum_{i=0}^{N} x_{ij}^{k} = y_{j}^{k}, \quad j = 0, 1, \ldots, N,\ k = 1, \ldots, K, \tag{2}$$

$$\sum_{k=1}^{K} y_{i}^{k} = \begin{cases} K, & i = 0 \\ 1, & i = 1, \ldots, N, \end{cases} \tag{3}$$

$$L \le \sum_{i=1}^{N} y_{i}^{k} q_{i} \le U, \quad k = 1, \ldots, K, \tag{4}$$

$$\sum_{j=1}^{N} Z_{ij}^{k} y_{j}^{k} q_{j} \le Q, \quad i = 1, \ldots, N,\ k = 1, \ldots, K, \tag{5}$$

$$x_{ij}^{k}, Z_{ij}^{k}, y_{i}^{k} \in \{0, 1\}_{i}, \quad i,j = 0, 1, \ldots, N, i \neq j,\ k = 1, \ldots, K. \tag{6}$$

where:

$$x_{ij}^{k} = 1,\ \text{if order } j \text{ is immediately rolled after order } i \text{ in the } k\text{th turn, and 0 otherwise,}$$
$$\text{for } i, j, = 0, 1, 2, \ldots, N,\ i \neq j;\ k = 1, \ldots, K,$$

$$y_{i}^{k} = 1,\ \text{if order } i \text{ rolled is in the } k\text{th turn, and 0 otherwise, for } i = 0, 1, 2, \ldots, N;\ k = 1, \ldots, K,$$

$$Z_{ij}^{k} = 1,\ \text{if order } j \text{ is immediately rolled after order } i \text{ in the } k\text{th turn, and order } i \text{ and } j \text{ are in the same width,}$$
$$\text{0 otherwise, for } i, j, = 0, 1, 2, \ldots, N, i \neq j, k = 1, \ldots, K,$$

The objective function, *Z1*, minimises the total penalty costs. Constraints (1) and (2) represent the assignment issues of CVRP. Constraint (3) ensures that an order is assigned to only one turn, and dummy node 0 is assigned to all turns. Constraint (4) enforces that the total length of each rolling turn has a lower limit and an upper limit, i.e. the total load of a turn (vehicle) is within a lower and upper limit. Constraint (5) is meant to enforce that the total length of continuously rolled orders with the same width is limited to a given value. Constraint (6) enforces binary conditions.

As Kim (2010) mentioned, constraint (5) does not represent 'the total length of continuously rolled orders with the same width'. Furthermore, Kim (2010) provided two corrections as follows:

Case 1: When all the segments of the continuously rolled orders are counted in 'the total length of the continuously rolled orders with the same width'.

Case 2: When only the maximum length of continuously rolled orders is counted in 'the total length of the continuously rolled orders with the same width'.

However, it is case 2 that is related to the SHRP. Thus, we address this case in our model, too. Although the correction by Kim is linear, it needs a subtour elimination to correctly solve the problem. We also use some of Kim's corrected inequalities to address case 2.

In VRP, each turn (vehicle) is to start from a dummy node 0 and ends up to it. If such constraints are not included, the solution becomes infeasible for the problem (Miller *et al*. 1960). Both models (Chen *et al*. 2008) and (Kim 2010) lack such subtour elimination. Below, we present a formulation of CVRP that corrects the CVRP formulation of Chen *et al*. (2008). The formulation is linear and substantially reduces the number of variables compared with Chen *et al*.'s and Kim's models.

Definition: Parameter: $s_{ij} = 1,$ if orders i and j have the same width, and 0 otherwise, for $i, j, = 0, 1, 2, \ldots, N, i \neq j.$

Decision variables:

$x_{ij} = 1,$ if order j is immediately rolled after order i in a turn, and 0 otherwise, for $i, j, = 0, 1, 2, \ldots, N,\ i \neq j,$
$u_i,$ cumulative length of rolled orders loaded on a vehicle starting from 0 to order i, for $i = 1, 2, \ldots, N,$
$t_i,$ cumulative length of continuously rolled orders with the same width after order i, for $i = 1, 2, \ldots, N.$

We generalise the CVRP formulation of Kara *et al*. (2004) and Kara and Bektas (2006) for our problem here. The formulation is as follows:

$$\text{Minimise} \quad Z2 = \sum_{j=1}^{N} \sum_{j=1}^{N} C_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} x_{0i} = \sum_{i=1}^{N} x_{i0} = K, \tag{7}$$

$$\sum_{j=0, j\neq i}^{N} x_{ij} = \sum_{j=0, j\neq i}^{N} x_{ji} = 1, \quad i = 1, \ldots, N, \tag{8}$$

$$u_i - u_j + U x_{ij} + \left(U - q_i - q_j\right) x_{ji} \leq U - q_j, \quad 1 \leq i \neq j \leq N, \tag{9}$$

$$q_i \leq u_i, \quad i = 1, \ldots, N, \tag{10}$$

$$u_i + (U - q_i) x_{0i} \leq U, \quad i = 1, \ldots, N, \tag{11}$$

$$u_i + (q_i - L) x_{i0} \geq q_i, \quad i = 1, \ldots, N, \tag{12}$$

$$t_i - t_j + Q s_{ij} x_{ij} \leq Q - q_j, \quad 1 \leq i \neq j \leq N, \tag{13}$$

$$q_i \leq t_i \leq Q, \quad i = 1, \ldots, N, \tag{14}$$

$$x_{ij} \in \{0, 1\}_i, \quad 1 \leq i \neq j \leq N, \tag{15}$$

$$u_i, t_i \geq 0, \quad 1 \leq i \leq N. \tag{16}$$

The objective function, $Z2$, minimises total penalty costs. We have $C_{ij} = P_{ij}^{w} + P_{ij}^{g} + P_{ij}^{H} + P_{ij}^{T}$ for $i, j = 1, \ldots, N,$ and $C_{ij} = 0$ for $i = 0, j = 1, \ldots, N,$ and $j = 0, i = 1, \ldots, N,$ where

$P_{ij}^{w},$ penalty value of the width jump from slab i to slab j,
$P_{ij}^{g},$ penalty value of the thickness jump from slab i to slab j,
$P_{ij}^{H},$ penalty value of the hardness jump from slab i to slab j,
$P_{ij}^{T},$ penalty value of the temperature jump from slab i to slab j.

Orders are known in advance. Thus, penalty values of $P_{ij}^{w}, P_{ij}^{g}, P_{ij}^{H}, P_{ij}^{T}$ can be chosen in advance and can be set such that the optimal solution of the problem guarantees satisfying conditions 1–3 of SHRP. This is the same way that

Chen *et al*. (2008) and Tang *et al*. (2000) handled constraints 1–3. Constraint (7) enforces each turn (vehicle) to start and end with dummy node 0. Constraint (8) is the usual balancing constraint for CVRP. Constraint (9) enforces a subtour elimination for each vehicle. These inequalities for all $1 \leq i \neq j \leq N$ ensure that $u_j = u_i + q_j$ if and only if $x_{ij} = 1$. Constraints (10)–(12) enforce the upper and lower bound for load of each turn. Here, if $x_{0i} = 1$, then we have $u_i = q_i$; and if $x_{i0} = 1$, then $L \leq u_i \leq U$. It allows for a vehicle i to carry only one job. In that case, $u_i = q_i \geq L$. Constraints (13) and (14) enforce the total length of the continuously rolled orders with the same width to be within the limit of Q. Constraints (15) and (16) enforce binary and continuous conditions, respectively. We refer readers to Kara *et al*. (2004) and Kara and Bektas (2006) for an extension of the Miller–Tucker–Zemlin subtour elimination for CVRP that we adopted here for CVRP in SHRP.

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